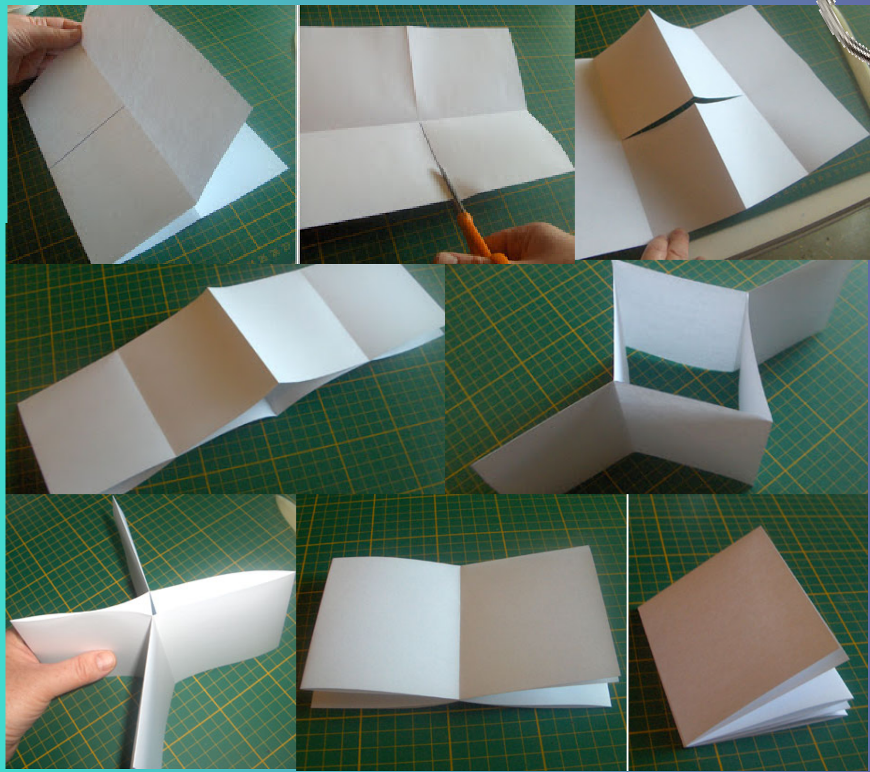
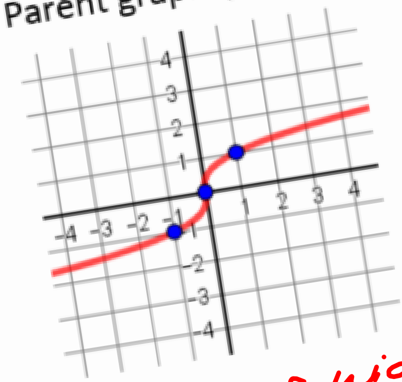


Graphing Cube Root Functions

Parent graph: $y = \sqrt[3]{x}$



Shape is a sideways wiggle

Center point at (0,0)

Passes through (1,1) and (-1,-1)

Domain: $\mathbb{R}$ Range: $\mathbb{R}$

increasing from left to right

Example 1 Describing transformations

Domain $\mathbb{R}$
Range $\mathbb{R}$

$\lvert a\rvert > 1$ vertical **stretch**
$\lvert a\rvert < 1$ vertical **shrink**
$a < 0$ **reflects** over **x-axis**

$+k$ translation **up**
$-k$ translation **down**

$$y = a\sqrt[3]{b(x-h)} + k$$

center (h, k)

$\lvert b\rvert > 1$ horizontal **shrink**
$\lvert b\rvert < 1$ horizontal **stretch**
$b < 0$ **reflects over y-axis**

$(x-h)$ translation **right**
$(x+h)$ translation **left**

*We won't look at horizontal dilations until PreCalc

Describe each graph as compared to the parent graph.

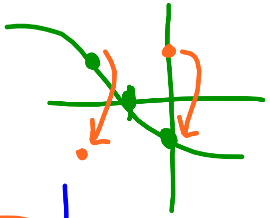
A] $y = \frac{1}{2}\sqrt[3]{x} + 4$	B] $y = -\sqrt[3]{x+5}$	C] $y = \sqrt[3]{-x+6}$	D] $y = 7\sqrt[3]{-x} - 8$
• vertical shrink of 1/2 • up 4 center $(0,4)$ D: $\mathbb{R}$ R: $\mathbb{R}$	• reflect in x-axis • left 5 center $(-5,0)$ D: $\mathbb{R}$ R: $\mathbb{R}$	$-(x-6)$ *warning! Factor out b-value first! • reflect in y-axis • right 6 center $(6,0)$ D: $\mathbb{R}$ R: $\mathbb{R}$	• vertical stretch 7 • reflect in y-axis • down 8 center $(0,-8)$ D: $\mathbb{R}$ R: $\mathbb{R}$

Describe the transformations of your function and sketch the graph on your whiteboard.

Talking Heads

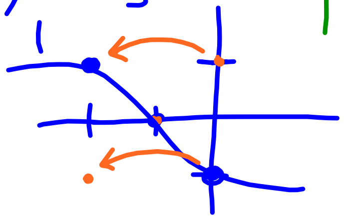
1's $y = -\sqrt[3]{x+1}$

reflect x-axis
left 1



2's $y = \sqrt[3]{-(x+1)}$

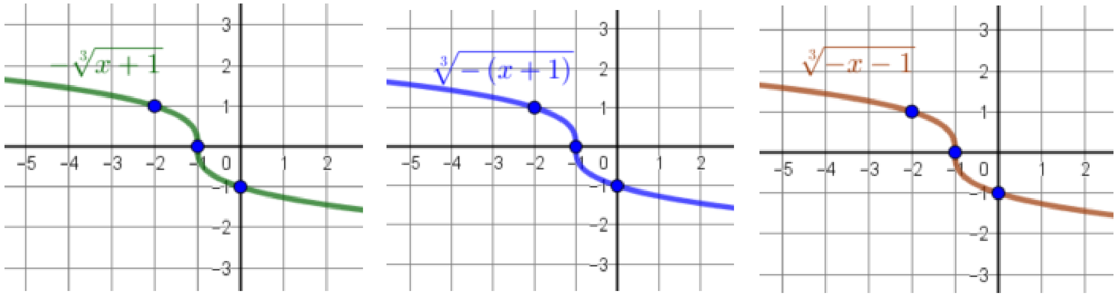
reflect y-axis
left 1



3's $y = \sqrt[3]{-x-1}$

$\sqrt[3]{-(x+1)}$

As a group, compare the transformations and graphs of each function.
Write a statement summarizing what you discovered during your discussion.

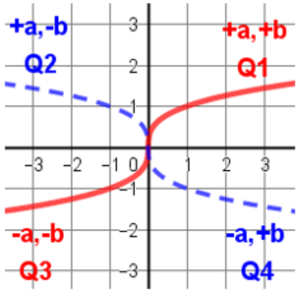


A reflection in the x-axis and a reflection in the y-axis produces the same graph.

Example 2 Graphing with transformations

Step 1: Plot the center point (h, k).

Step 2: Use the signs of *a* and *b* to decide which quadrants the curve is heading toward, then use $\pm a/1$ to find the guide points in those directions.



Step 3: Substitute zero for x to find the y-intercept. Round to one decimal place when needed.

Step 4: Find more guide points. Pick x's that make perfect cubes to make mental math easy. Use a calculator to approximate cube roots if needed.

A] $y = -6\sqrt[3]{x-2} + 10$

Center point (h, k): (2,10)

Heading toward: Q2 and 4

Guide points using $\pm a/1$:

(1,16) and (3,4)

y-intercept: (0, 17.6)

$-6\sqrt[3]{0-2}+10 \approx$

```
-6(∛(0-2))+10
      17.5595263
```

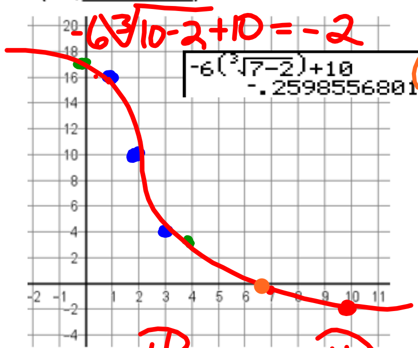
Extra guide point(s):

(10, -2)

$-6\sqrt[3]{10-2}+10 = -2$

```
-6(∛(7-2))+10
   -.2598556801
```

(7,-0.3)



Domain: $\mathbb{R}$ Range: $\mathbb{R}$

B] $y = -\frac{1}{2}\sqrt[3]{-x-3} - 1$

$-0.5\sqrt[3]{-(x+3)} - 1$

Center point (h, k): (-3,-1)

Heading toward: Q1 and 3

Guide points using $\pm a/1$:

$(-4, -\frac{3}{2})$ and $(-2, -\frac{1}{2})$

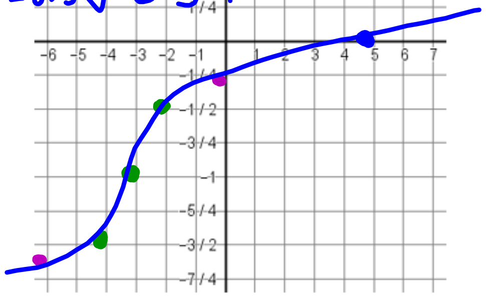
y-intercept: (0, -0.3)

$-0.5\sqrt[3]{0-3}-1 \approx$

```
-.5(∛(-0-3))-1
   -.2788752148
```

Extra guide point(s): (5, 0)

$-0.5\sqrt[3]{-5-3}-1 = 0$



Domain: $\mathbb{R}$ Range: $\mathbb{R}$

On your whiteboard...

$$y = 9\sqrt[3]{-x+3} - 6 \qquad 9\sqrt[3]{-(x-3)}-6$$

Center point (h, k): $(3, -6)$

Heading toward: Q2 and 4

Guide points using $\pm a/1$:

$(2, 3)$ and $(4, -15)$

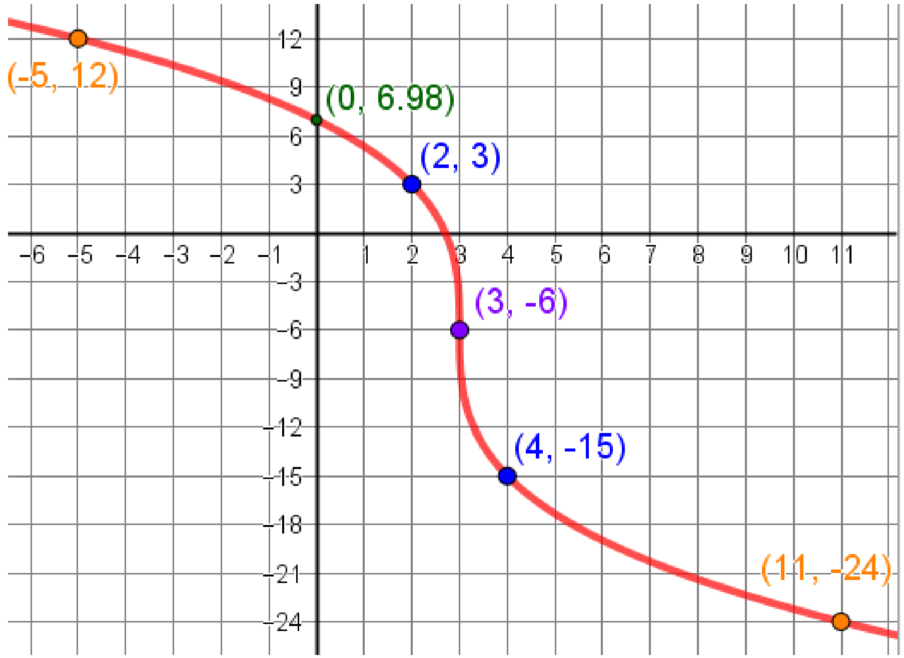
y-intercept: $(0, 6.98)$

Pick x's that make a perfect cube

*Extra guide point(s):

$(-5, 12)$ and $(11, -24)$

Domain: $\mathbb{R}$ Range: $\mathbb{R}$

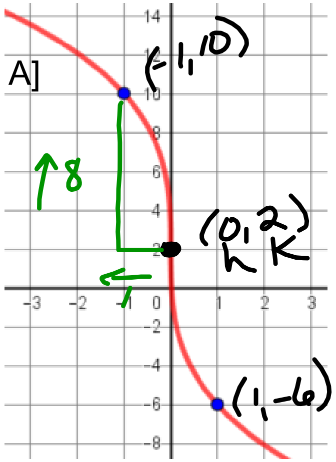


Example 3 Write the equation from the graph

$$y = a\sqrt[3]{x-h}+k$$

The center point is (h,k) and $\pm a$ is the slope from the endpoint to the guide points exactly one point to the right and left of the center point. Since a reflection over the x-axis is the same graph as a reflection over the y-axis, just make a negative if the graph is decreasing from left to right.

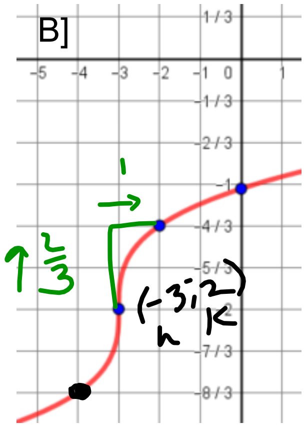
A]



$h=0$ $k=2$ $a=-8$

$y=-8\sqrt[3]{x}+2$

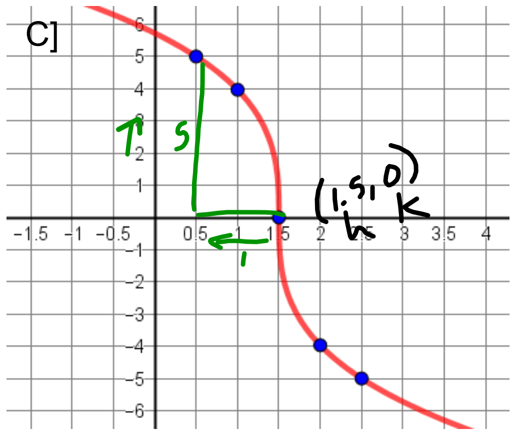
B]



$h=-3$ $k=-2$ $a=\frac{2}{3}$

$y=\frac{2}{3}\sqrt[3]{x+3}-2$

C]



$h=1.5$ $k=0$ $a=-5$

$y=-5\sqrt[3]{x-1.5}$

Example 4 Write the equation from characteristics

Write the equation for a radical function that is centered at (-3,-12) and passes through (24,-13).

h k

x y

$$y = a\sqrt[3]{x-h}+k$$

Use these given values in the equation to solve for *a*.

$h=-3$
$k=-12$
$x=24$
$y=-13$

$-13 = a\sqrt[3]{24-(-3)}+(-12)$

$-13 = a\sqrt[3]{27}-12$

$+12 \qquad\qquad +12$

$\frac{-1}{3} = \frac{3a}{3}$

$-\frac{1}{3} = a \rightarrow y = -\frac{1}{3}\sqrt[3]{x+3}-12$