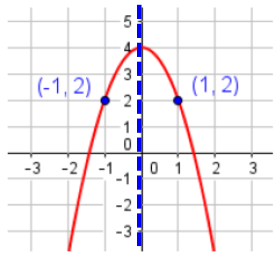
Math Lab: Common Types of Symmetry

y-axis symmetry

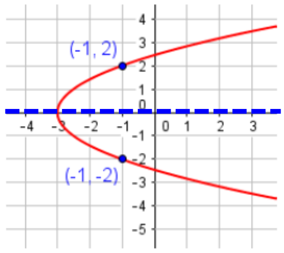
If you fold the graph along the y-axis, the sections of the graph left and right of the axis would coincide.



If (x, y) is on the graph, so is the point (**-x** , **y**)

x-axis symmetry

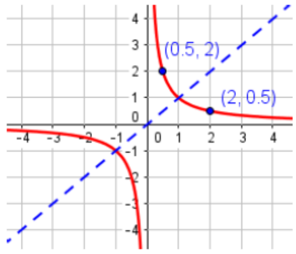
If you fold the graph along the x-axis, the sections of the graph above and below of the axis would coincide.



If (x, y) is on the graph, so is the point (**x** , **-y**)

symmetry about the line y = x

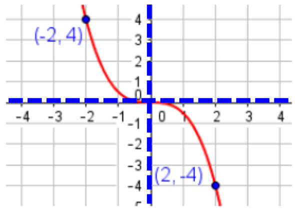
If you fold the graph along the line $y = x$, the sections of the graph above and below the line would coincide.



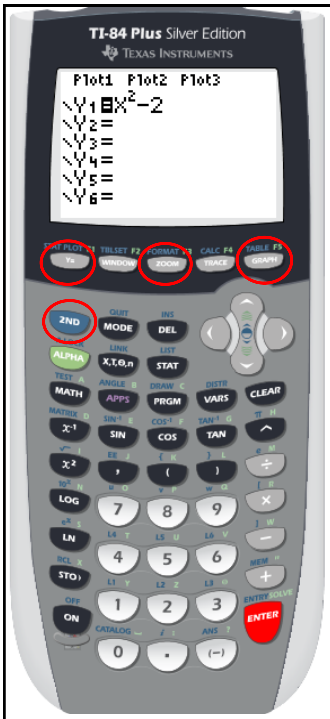
If (x, y) is on the graph, so is the point (**y** , **x**)

Origin (rotational) symmetry

If you rotate the graph 180 degrees, you get the same graph again. Origin symmetry is the same as reflecting a graph over the x-axis and y-axis.



If (x, y) is on the graph, so is the point (**-x** , **-y**)



You can determine symmetry of a given equation using a graphing calculator by viewing the table or the graph.

Determine the type of symmetry, if any, of the graph $y = x^2 - 2$

Viewing the table:

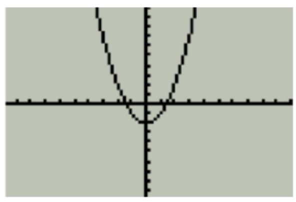
Solve the equation for y on the left, if needed. Enter into Y= and press 2^{nd} GRAPH to view the table. Check that for each point (x, y), the point (-x, y) is also in the table.

X	Y1
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

For example, the point (-3, 7) is in the table. Verify then that the point (3, 7) is also in the table to conclude it has y-axis symmetry.

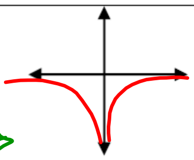
Viewing the graph:

Solve the equation for y on the left, if needed. Enter into Y= and press ZOOM 6 to see the graph. You might need to ZOOM IN or ZOOM OUT.



Observe the graph and notice that it appears to have y-axis symmetry because if you fold the graph along the y-axis, the two halves meet up.

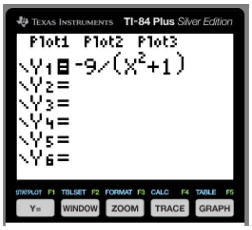
Practice: Use a graphing calculator or Desmos to make a quick sketch to show the shape of the graph. Use the graph and the table of ordered pairs to determine the type of symmetry. For each type of symmetry the function has, identify a set of ordered pairs from the table that demonstrates that type of symmetry and record them in the corresponding box. Leave the other boxes empty. Some graphs will have more than one type of symmetry!

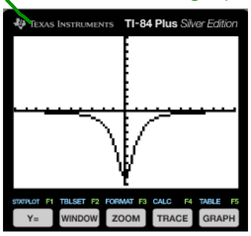
Equation	Quick Sketch	y-axis symmetry (x,y) (-x,y)	x-axis symmetry (x,y) (x,-y)	y = x symmetry (x,y) (y,x)	origin symmetry (x,y) (-x,-y)	no symmetry
1. $y = \frac{-9}{x^2+1}$ $Y1 = -9/(x^2+1)$		(-3, -0.9) (3, -0.9)				

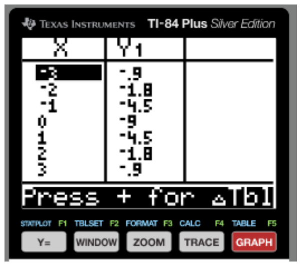
Write what you type into the calculator here.

Zoom in if needed, then sketch the graph.

Find a set of ordered pairs that shows the symmetry and write in the corresponding box(es).



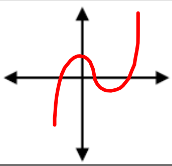


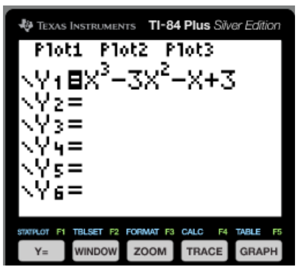


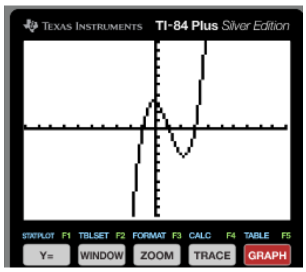
How will you enter this equation into the calculator?

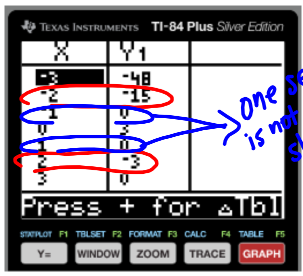
* Solve for y first.

$$x^3+3 = y+x+3x^2$$
$$-x \; -3x^2 \qquad -x \; -3x^2$$
$$x^3-3x^2-x+3 = y$$

Equation	Quick Sketch	y-axis symmetry (x,y) (-x,y)	x-axis symmetry (x,y) (x,-y)	y = x symmetry (x,y) (y,x)	origin symmetry (x,y) (-x,-y)	no symmetry
2. $x^3 + 3 = y + x + 3x^2$ $y1 = x^3 - 3x^2 - x + 3$						(-2, -15) (2, -3)







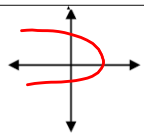
One set of points is not enough to show y-axis symmetry. Must be true for all.

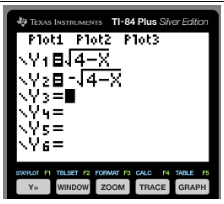
Turn & Talk

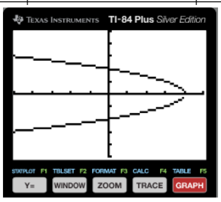
Why does $y^2 + x = 4$ have to be entered in two parts as $y_1 = \sqrt{4-x}$, $y_2 = -\sqrt{4-x}$?

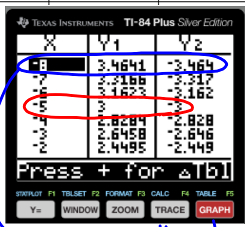
When you solve for y you get two equations:

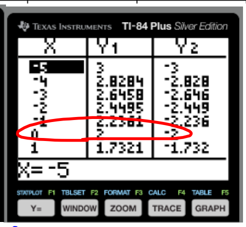
$$y^2 + x = 4$$
$$\quad -x \quad -x$$
$$\sqrt{y^2} = \sqrt{4-x}$$
$$y = \pm\sqrt{4-x} \;\rightarrow\; y = \sqrt{4-x},\quad y = -\sqrt{4-x}$$

Equation	Quick Sketch	(x,y) (-x,y) y-axis symmetry	(x,y) (x,-y) x-axis symmetry	(x,y) (y,x) y = x symmetry	(x,y) (-x,-y) origin symmetry	no symmetry
5. $y^2 + x = 4$ Hint: enter it in two parts… $y_1 = \sqrt{4-x}$ $y_2 = -\sqrt{4-x}$			(-5,3) (-5,-3) or (0,2) (0,-2)			







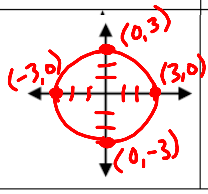


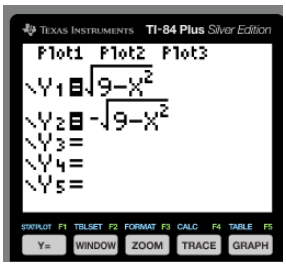
When equations have radicals ($\sqrt{x}$ or $\sqrt[3]{x}$ etc) the y-values are irrational numbers and the calculator often rounds decimals differently. So for irrational numbers, scroll to find integer y-values and ignore decimals.

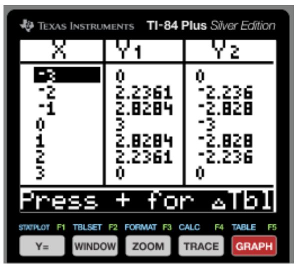
Turn & Talk

How many types of symmetry does this equation have?

$x^2+y^2=9$ is a circle of radius 3 centered at the origin, so it has all 4 types of symmetry.

Equation	Quick Sketch	(x,y) (-x,y) y-axis symmetry	(x,y) (x,-y) x-axis symmetry	(x,y) (y,x) y = x symmetry	(x,y) (-x,-y) origin symmetry	no symmetry
8. $x^2 + y^2 = 9$ Hint: enter it in two parts… $y_1 = \sqrt{9-x^2}$ $y_2 = -\sqrt{9-x^2}$		(-3,0) (3,0)	(0,3) (0,-3)	(0,3) (3,0)	(-3,0) (3,0)	





- Finish the rest on your own.
- Check answers with your partner as you go.
- Check your answers with at least THREE more classmates before you turn in the assignment.

Practice: Use a graphing calculator or Desmos to make a quick sketch to show the shape of the graph. Use the graph and the table of ordered pairs to determine the type of symmetry. For each type of symmetry the function has, identify a set of ordered pairs from the table that demonstrates that type of symmetry and record them in the corresponding box. Leave the other boxes empty. Some graphs will have more than one type of symmetry!

Equation	Quick Sketch	y-axis symmetry	x-axis symmetry	y = x symmetry	origin symmetry	no symmetry
1. $y = \frac{-9}{x^2+1}$						
2. $x^3 + 3 = y + x + 3x^2$						
3. $y = 3\sqrt[3]{x}$						
4. $y = x^4 - 6x^2 + 6$						
5. $y^2 + x = 4$ Hint: enter it in two parts… $y_1 = \sqrt{4-x}$ $y_2 = -\sqrt{4-x}$						
6. $y = 2\sqrt[3]{\lvert 4x\rvert - x^2}$						
7. $y = -x$						
8. $x^2 + y^2 = 9$ Hint: enter it in two parts… $y_1 = \sqrt{9-x^2}$ $y_2 = -\sqrt{9-x^2}$						