

Transformations

Translation: shift or move

Reflection: flip or mirror image

Dilation: stretch or shrink

Rigid: same shape/size but different position

Non-rigid: same shape but different size

Example 1 Identify transformations from the equation

[A] $f(x) = 7 + \frac{1}{2}(x+1)^2 - 10$

Standard form: $f(x) = \frac{1}{2}(x+1)^2 - 3$

Type of function: Quadratic

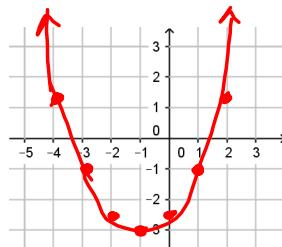
Rigid: down 3
left 1

dilation $\frac{1}{2}$
Non-rigid:

Guide points

$$f(1) = -1$$

$$f(2) = 1.5$$



[B] $f(x) = \frac{2}{x-3} + 1$

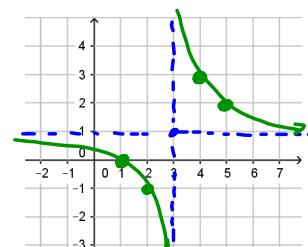
Standard form: $f(x) = 2\left(\frac{1}{x-3}\right) + 1$

Type of function: Rational

Rigid: up 1
right 3

dilation 2
Non-rigid:

Guide points
 $f(5) = 2$
 $f(1) = 0$



On your whiteboard...

Check all that apply and fill-in blanks

[A] $f(x) = -3 + \frac{1}{2}(1+x)^2$ $\frac{1}{2}(x+1)^2 - 3$

Type: Quadratic

vertical translation up ____ units

vertical translation down 3 units

horizontal translation right ____ units

horizontal translation left 1 units

reflection in the ____ axis

dilation of $\frac{1}{2}$; vertical shrink

dilation of ____ ; horizontal _____

[B] $f(x) = 2\sqrt{-(x-3)} - 4$

Type: Square Root

vertical translation up ____ units

vertical translation down 4 units

horizontal translation right 3 units

horizontal translation left ____ units

reflection in the y axis

dilation of 2; vertical Stretch

dilation of ____ ; horizontal _____

On your whiteboard...

Check all that apply and fill-in blanks

$$f(x) = 2 - \sqrt{x+1} + 7 = -\sqrt{x+1} + 7 + 2 = -\sqrt{x+1} + 9$$

Type: Square Root

vertical translation up 9 units

vertical translation down ____ units

horizontal translation right ____ units

horizontal translation left 1 units

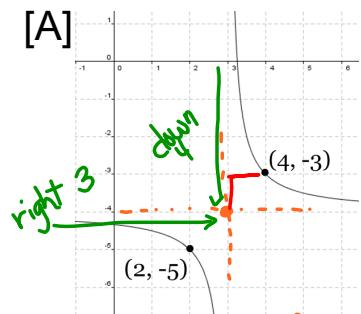
reflection in the X axis

dilation of ____ ; vertical _____

dilation of ____ ; horizontal _____

Example 2 Write the equation for the graph.

[A]



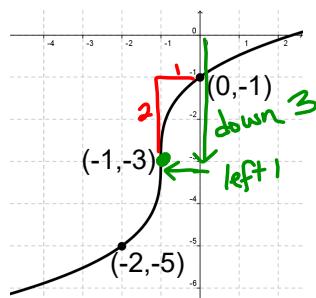
Type of function: Rational $\frac{1}{x}$

Rigid: down 4
right 3

Non-rigid: none

Equation: $f(x) = \frac{1}{x-3} - 4$

[B]



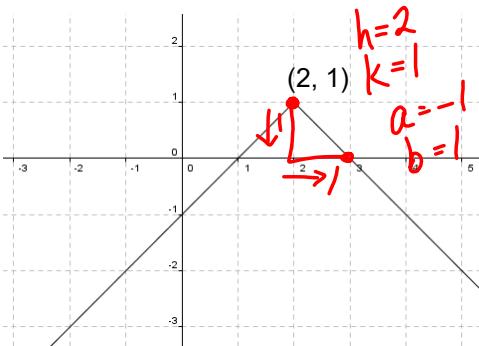
Type of function: Cube Root $\sqrt[3]{x}$

Rigid: down 3
left 1

Non-rigid: dilation 2

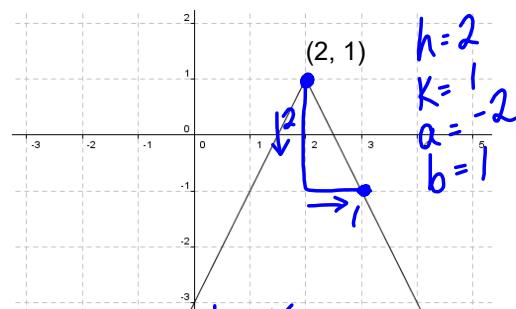
Equation: $f(x) = 2\sqrt[3]{x+1} - 3$

On your whiteboard... Write the equation for the graph.



$f(x) = -|x-2| + 1$

reflected x-axis
right 2
up 1



$f(x) = -2|x-2| + 1$

reflected x-axis
vertically stretch 2
right 2
up 1

Example 3 Write the equation given a description

On the back!

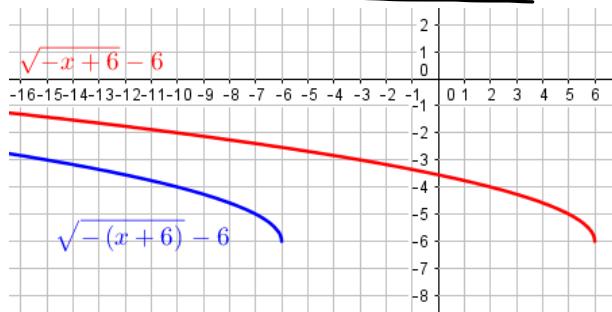
A function with a domain of $(-\infty, -6]$ and a range of $[-6, \infty)$ that has no non-rigid transformations.

Only function with half bounded domain & range is square root!
 $y = \sqrt{x}$

$a = -1$ * reflected in the y-axis
 $h = -6$ * left 6
 $k = -6$ * down 6
 $b = 1$ * no dilations

be careful mixing y-axis reflections with horizontal shifts ✗

$$f(x) = \sqrt{-(x+6)} - 6$$

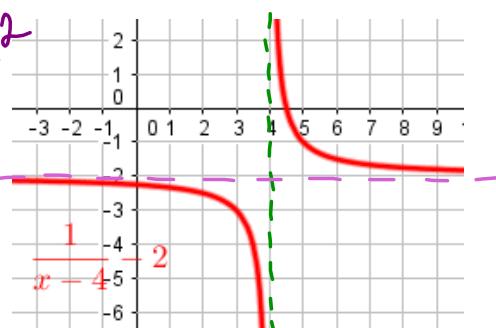


On your whiteboard... Write the equation from the description

A function with a domain of $(-\infty, 4) \cup (4, \infty)$ and a range of $(-\infty, -2) \cup (-2, \infty)$ that has not been reflected or dilated. ↗ Rational!

$$f(x) = \frac{1}{x-4} - 2$$

no reflection or dilation ↗
 right 4 ↗
 down 2 ↗

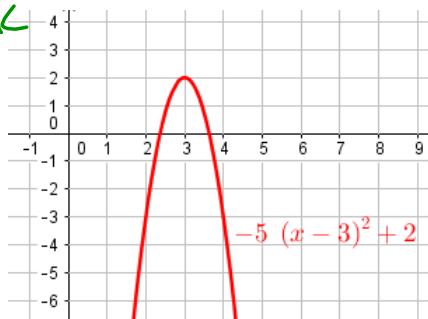


On your whiteboard... Write the equation from the description

A function with a range of $(-\infty, 2]$ with a vertex that has an x-coordinate of 3. It graphs a smooth curve that has been vertically stretched by a factor of 5.

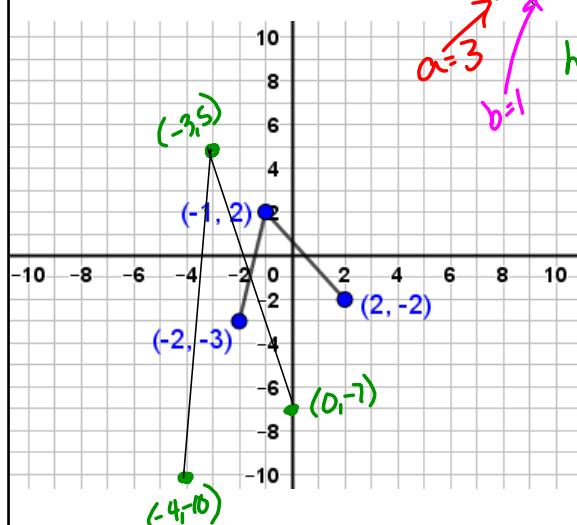
$$f(x) = -5(x-3)^2 + 2$$

reflected over x-axis
vertical stretch of 5
right 3
up 2
Quadratic



Example 4 Sketch a graph using transformations

Use transformations to sketch $3f(x+2) - 1$ **Must follow the order of operations (mult/div before add/sub)



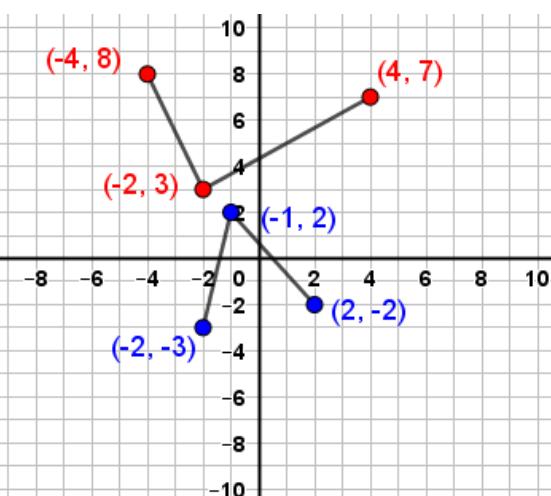
$$a=3 \quad h=-2 \quad k=-1$$

$$a \cdot f(b(x-h)) + k$$

Three anchor points	Multiply y-value by $a=3$	Divide x-value by $b=1$	Add $h=-2$ to the x-value	Add $k=-1$ to the y-value
$(-1, 2)$	$(-1, 6)$	\longrightarrow	$(-3, 6)$	$(-3, 5)$
$(-2, -3)$	$(-2, -9)$	\longrightarrow	$(-4, -9)$	$(-4, -10)$
$(2, -2)$	$(2, -6)$	\longrightarrow	$(0, -6)$	$(0, -7)$

Try this one on your own...

Use transformations to sketch $-f\left(\frac{1}{2}x\right) + 5$



$$a = -1 \quad b = \frac{1}{2} \quad h = 0 \quad k = 5$$

$$a \cdot f(b(x - h)) + k$$

Three anchor points	Multiply y-value by $a = -1$	Divide x-value by $b = \frac{1}{2}$	Add $h = 0$ to the x-value	Add $k = 5$ to the y-value
(-1, 2)	(-1, -2)	(-2, -2)	→	(-2, 3)
(-2, -3)	(-2, 3)	(-4, 3)	→	(-4, 8)
(2, -2)	(2, 2)	(4, 2)	→	(4, 7)

On your whiteboard...

[A] If $(3, 7)$ is a point on the graph of $f(x)$,

what point is on the graph of $2f(x - 3) + 5$?

$$(3, 7) \xrightarrow{\text{dilate } 2} (3, 14) \xrightarrow{\text{right } 3} (6, 14) \xrightarrow{\text{up } 5} (6, 19)$$

[B] If $(-2, -1)$ is a point on the graph of $f(x)$,

what point is on the graph of $-f(x + 2) - 4$?

$$(-2, -1) \xrightarrow{\text{reflect } x\text{-axis}} (-2, 1) \xrightarrow{\text{left } 2} (-4, 1) \xrightarrow{\text{down } 4} (-4, -3)$$

[C] If $(6, -4)$ is a point on the graph of $f(x)$,

what point is on the graph of $\frac{1}{2}f(-x) + 3$?

$$(6, -4) \xrightarrow{\text{dilate } y2} (6, -2) \xrightarrow{\text{reflect } y\text{-axis}} (-6, -2) \xrightarrow{\text{up } 3} (-6, 1)$$