

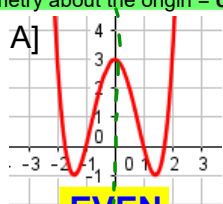
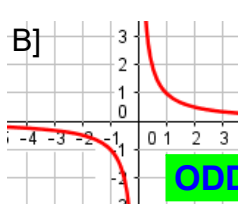
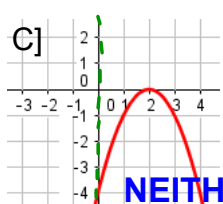
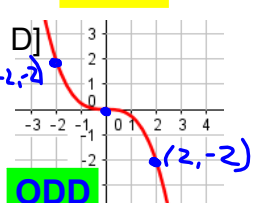




# Even & Odd Functions

**y-axis symmetry: EVEN**  
 Mirror image over y-axis  
 $(x,y) \rightarrow (-x,y)$   
 $f(-x) = f(x)$

**origin (rotational) symmetry: ODD**  
 Centered at the origin and rotating  $180^\circ$  is same graph  
 Same as reflecting in both axes  
 $(x,y) \rightarrow (-x,-y)$   
 $f(-x) = -f(x)$

## Example 1 Using the graph

|  | EVEN        | ODD        | NEITHER  |
|--|-------------|------------|--|
| When looking at the graph, first make sure it is a function, then...                     |             |            |  |
| Symmetry about the y-axis = even function  |             |            |  |
| Symmetry about the origin = odd function   |             |            |  |
| A]    | <b>EVEN</b> |            |  |
| B]    |             | <b>ODD</b> |  |
| C]  |             |            | <b>NEITHER</b><br>axis of symmetry is $x=2$ , not y-axis |
| D]    |             | <b>ODD</b> |  |
| E]    |             |            | <b>NEITHER</b><br>not a function, fails the VLT          |
| F]  |             |            | <b>NEITHER</b><br>not a function, fails the VLT          |

## Example 2 Using a table of ordered pairs

When looking at a table...

**EVEN**

**ODD**

**NEITHER**

If  $(x, y)$  and  $(-x, y)$  are in the table = **even function**

If  $(x, y)$  and  $(-x, -y)$  are in the table = **odd function**

A)

| x  | y     |
|----|-------|
| -3 | -2/3  |
| -2 | -1    |
| -1 | -2    |
| 0  | error |
| 1  | 2     |
| 2  | 1     |
| 3  | 2/3   |

$(x, y)$  and  $(-x, -y)$  are circled in red. **ODD**

B)

| x  | y  |
|----|----|
| -3 | 10 |
| -2 | 4  |
| -1 | 0  |
| 0  | -2 |
| 1  | -2 |
| 2  | 0  |
| 3  | 4  |

$(-3, 10)$  and  $(3, 4)$  are circled in red. **NEITHER**

C)

| x  | y  |
|----|----|
| -6 | 69 |
| -4 | 29 |
| -2 | 5  |
| 0  | -3 |
| 2  | 5  |
| 4  | 29 |
| 6  | 69 |

$(-6, 69)$  and  $(6, 69)$  are circled in red. **EVEN**

On your whiteboard...

Identify each as even, odd, or neither.

[A]

| X  | Y1    |
|----|-------|
| -3 | -13.5 |
| -2 | -4    |
| -1 | -1.5  |
| 0  | 0     |
| 1  | 1.5   |
| 2  | 4     |
| 3  | 13.5  |

Press + for  $\Delta$ tbl

**ODD**

| X  | Y1    |
|----|-------|
| -3 | -13.5 |
| -2 | -4    |
| -1 | -1.5  |
| 0  | 0     |
| 1  | 1.5   |
| 2  | 4     |
| 3  | 13.5  |

Press + for  $\Delta$ tbl

[B]

| X    | Y1     |
|------|--------|
| -1.5 | -.9844 |
| -1   | -.75   |
| -.5  | -.2344 |
| 0    | 0      |
| .5   | -.2344 |
| 1    | -.75   |
| 1.5  | -.9844 |

X=-1.5

**EVEN**

| X    | Y1     |
|------|--------|
| -1.5 | -.9844 |
| -1   | -.75   |
| -.5  | -.2344 |
| 0    | 0      |
| .5   | -.2344 |
| 1    | -.75   |
| 1.5  | -.9844 |

X=-1.5

[C]

| X  | Y1 |
|----|----|
| -3 | 16 |
| -2 | 9  |
| -1 | 4  |
| 0  | 1  |
| 1  | 0  |
| 2  | 1  |
| 3  | 4  |

X=-3

**NEITHER**

| X  | Y1 |
|----|----|
| -3 | 16 |
| -2 | 9  |
| -1 | 4  |
| 0  | 1  |
| 1  | 0  |
| 2  | 1  |
| 3  | 4  |

X=-3

[D]

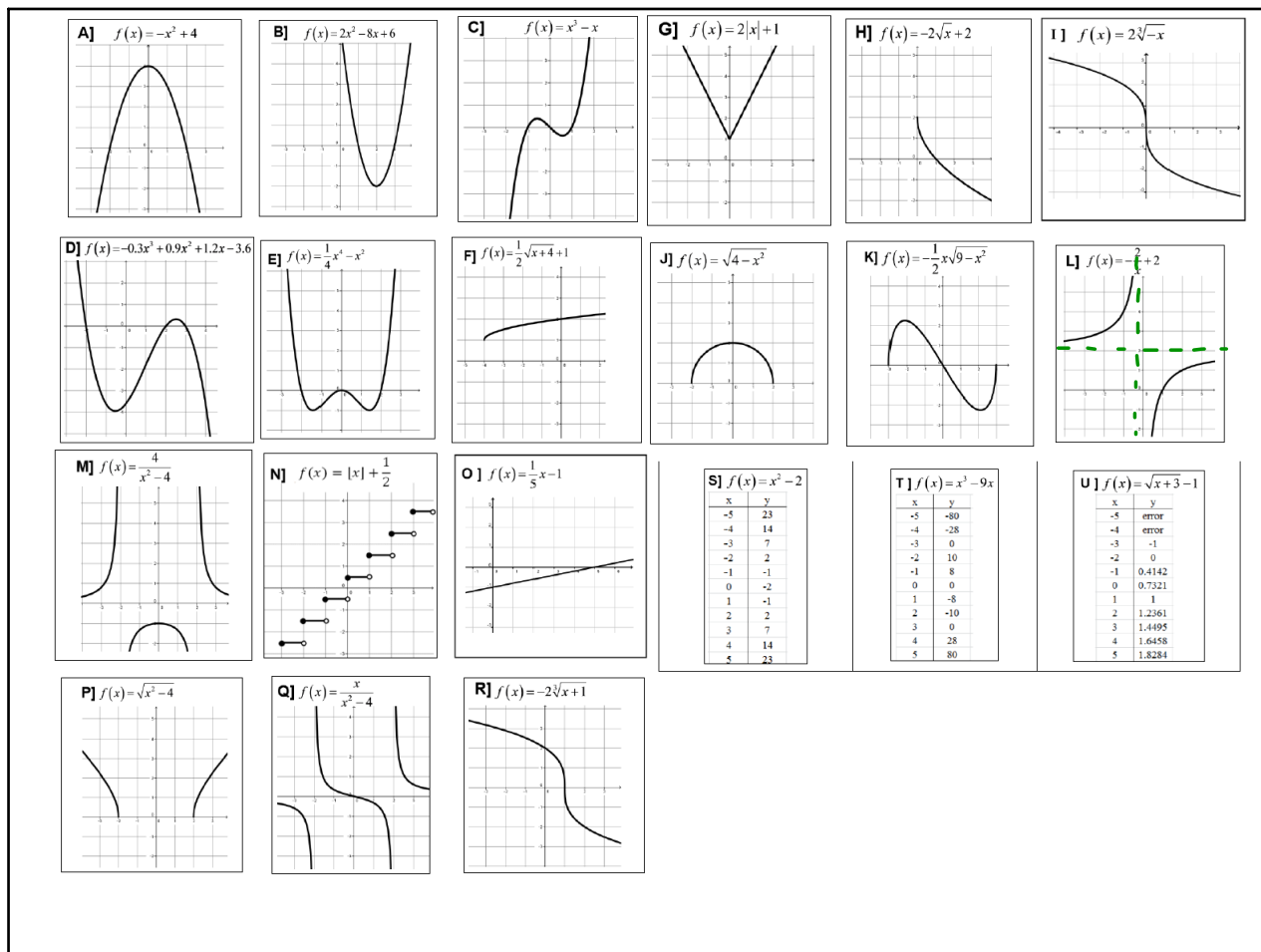
| X  | Y1     |
|----|--------|
| -6 | 5.6569 |
| -4 | 3.4641 |
| -2 | 0      |
| 0  | ERROR  |
| 2  | 0      |
| 4  | 3.4641 |
| 6  | 5.6569 |

X=-6

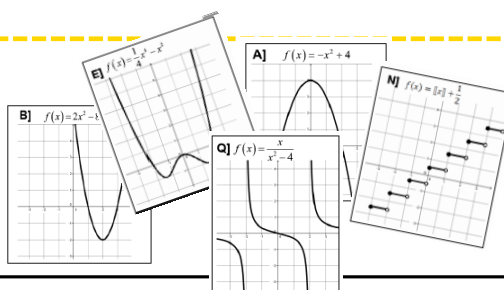
**EVEN**

| X  | Y1     |
|----|--------|
| -6 | 5.6569 |
| -4 | 3.4641 |
| -2 | 0      |
| 0  | ERROR  |
| 2  | 0      |
| 4  | 3.4641 |
| 6  | 5.6569 |

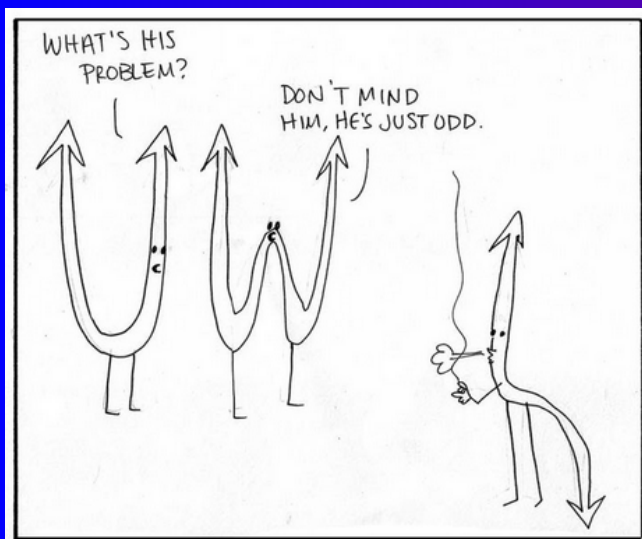
X=-6



Sort your graph cards into the following categories:



| Even functions   | Odd functions | Neither                 |
|------------------|---------------|-------------------------|
| A E G J<br>M P S | C I K<br>Q T  | B D F H<br>L N O R<br>U |



**LOL!**

Why is this cartoon funny?

Discuss with your partner and write your explanation on your white board, using as MANY different math vocabulary words as you can!

Be ready to share with the class.

The quadratic function's graph is a parabola, so if its vertex was on the y-axis it would have y-axis symmetry and be an even function. The polynomial standing next to the parabola would also have y-axis symmetry if its center was along the y-axis, so it would be an even function as well. The two even functions are talking to each other about the cubic function whose graph is a wiggle shape that has been reflected over either the x-axis or y-axis since it is decreasing from left to right. If the cubic were centered at the origin, it would be an odd function. The parabola is asking "What's his problem?" because the cubic has different symmetry, different end behavior, and has a reflection which the others don't have. Another difference is that the cubic is always decreasing from left to right, while the other graphs with him have intervals of increasing and decreasing. In a non-math sense, the cubic is different because he is smoking by himself while the others are not. The 4th degree polynomial says to the quadratic "Don't mind him, he's just odd," which in a non-math sense means that he is different from them, but in a math sense means that if the cubic were centered at the origin, it would be an odd function due to its rotational symmetry around the origin.

### Example 3 Verifying algebraically

#### Even functions:

$$f(-x) = f(x)$$

Substitute  $-x$  in the function and simplify. If you get back the original function, it is even.

#### Odd functions:

$$f(-x) = -f(x)$$

If you get back the opposite of the function, it is odd.

means you must write a proof to show it is even, odd, or neither using the algebraic definitions!

A)  $f(x) = 4x^2 - 10$

| Statements             | Reasons                           |
|------------------------|-----------------------------------|
| $f(x) = 4x^2 - 10$     | Given                             |
| $f(-x) = 4(-x)^2 - 10$ | Substitute $-x$ for $x$           |
| $f(-x) = 4x^2 - 10$    | Simplify *Goal is to match $f(x)$ |
| $f(-x) = f(x)$         | Definition of <u>EVEN</u>         |

B)  $f(x) = -9\sqrt[3]{x}$

| Statements                 | Reasons                            |
|----------------------------|------------------------------------|
| $f(x) = -9\sqrt[3]{x}$     | Given                              |
| $f(-x) = -9\sqrt[3]{-x}$   | Substitute $-x$ for $x$            |
| $f(-x) = -(-9\sqrt[3]{x})$ | Simplify *Goal is to match $-f(x)$ |
| $f(-x) = 9\sqrt[3]{x}$     |                                    |
| $f(x) = f(x)$              | Definition of <u>ODD</u>           |

C)  $f(x) = |x - 5|$

| Statements          | Reasons                                      |
|---------------------|--|
| $f(x) =  x - 5 $    | Given  |
| $f(-x) =  -x - 5 $  | Substitute $-x$ for $x$                      |
| $f(x) =  -(x + 5) $ | Simplify *Goal is to match $f(x)$ or $-f(x)$ |
| $f(x) =  x + 5 $    | Definition of                                |
| $f(-x) \neq f(x)$   |  |
| $f(-x) \neq -f(x)$  | <u>NEITHER</u>                               |

On your whiteboard...

Verify algebraically that  $f(x)$  is even, odd, or neither

A]  $f(x) = -\frac{6}{x}$

| Statement                 | Reason  |
|---------------------------|---|
| $f(x) = -\frac{6}{x}$     | Given   |
| $f(-x) = \frac{-6}{(-x)}$ | Substitute $-x$ for $x$                                   |
| $f(-x) = -(-\frac{6}{x})$ | Simplify<br>*Goal is to make it match $f(x)$ or $-f(x)$ . |
| $f(-x) = f(x)$            | Definition of <b>ODD</b>                                  |

B]  $f(x) = x^4 - 6x^2 + 6$

| Statements                     | Reasons   |
|--------------------------------|---|
| $f(x) = x^4 - 6x^2 + 6$        | Given   |
| $f(-x) = (-x)^4 - 6(-x)^2 + 6$ | Substitute $-x$ for $x$                                   |
| $f(-x) = x^4 - 6x^2 + 6$       | Simplify<br>*Goal is to make it match $f(x)$ or $-f(x)$ . |
| $f(-x) = f(x)$                 | Definition of <b>EVEN</b>                                 |

#### Example 4 Shortcut for polynomials

EVEN polynomials have ALL EVEN exponents.

ODD polynomials have ALL ODD exponents.

Only works for polynomials  
 $ax^n + bx^{n-1} + \dots + c$   
 the exponents are always positive whole numbers!

$c(x) = x^6 - 2x^2 + 3x^0$   
**EVEN**

$f(x) = x^6 - 3x^0$   
**EVEN**

$d(x) = x^3 - 3x^0$   
**NEITHER**

$e(x) = 4x^5 - 2x^3 + 3x^1$   
**ODD**

## DISCUSSION: Even, Odd, or Neither



What is the most important thing you need to remember about classifying functions as even, odd, or neither?

1] Answer the question in complete sentences giving as much detail as possible and using math vocabulary correctly. There is an equation editor in the tool bar you can use when entering equations and you can include images if you'd like to by copy/pasting the image into your post.

2] Post a comment on two other answers from your classmates. Do NOT leave a comment on an answer that already has two comments attached. Your comment should explain why you agree or disagree with their post and provide additional information regarding even, odd, or neither functions.

Posted Wed Sep 12, 2018 at 6:42 pm

| Criteria  | Grading Scale                              |  |                        |                        |               |
|---|--|--|------------------------|------------------------|---------------|
| <b>Your post</b><br>Answer to the discussion question is in detailed, complete sentences that demonstrate understanding of the topic and use appropriate vocabulary and use of notation.  | 4<br>Excellent                             | 3<br>Good  | 2<br>Satisfactory      | 1<br>Needs Improvement | 0<br>Not done |
| <b>Comment #1</b><br>Comment to classmate is left on a post that does not already have two comments attached. Comment explains in detail why you agree or disagree with the post and provides additional information about the topic. | 3<br>Good comment meeting all requirements | 2<br>Good comment but left for a classmate who already has two comments attached | 1<br>Needs Improvement | 0<br>Not done          |               |
| <b>Comment #2</b><br>Comment to classmate is left on a post that does not already have two comments attached. Comment explains in detail why you agree or disagree with the post and provides additional information about the topic. | 3<br>Good comment meeting all requirements | 2<br>Good comment but left for a classmate who already has two comments attached | 1<br>Needs Improvement | 0<br>Not done          |               |