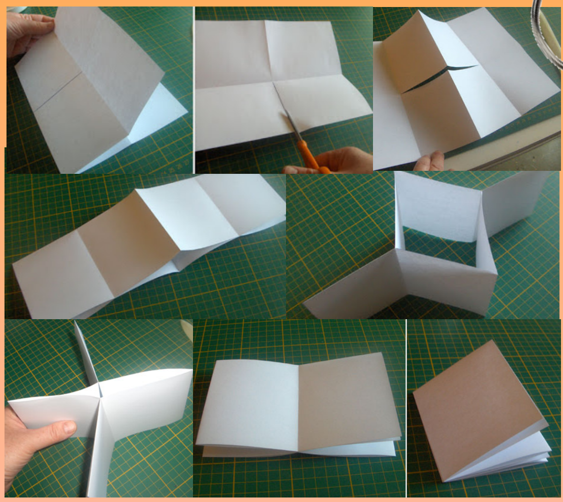


Piecewise Functions

- A function defined by multiple sub-functions, each sub-function applying to a certain interval of the main function's domain (called a subdomain)
- The pieces can be any type of function; try using transformations to graph
- Use open/closed circles to indicate if endpoints are included in the domain
- Can be continuous or discontinuous

Example 1 Evaluating Piecewise Functions

Determine which piece is used based on the domain restrictions given.
Substitute x into that piece and evaluate for y.

A]

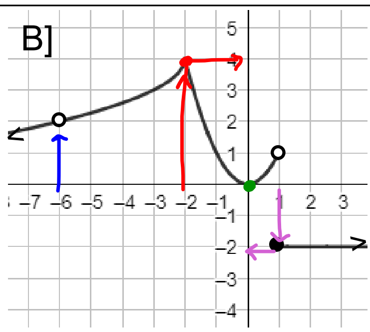
$$f(x)=\begin{cases} \boxed{x+4} & x<-2 \\ \boxed{\frac{1}{2}x^2} & -2\le x\le 2 \\ \boxed{-2\sqrt{x-2}+1} & x>2 \end{cases}$$

$f(6) = -2\sqrt{6-2}+1 = 2\sqrt{4}+1 = 5$

$f(-12) = -12+4 = -8$

$f\left(\frac{3}{2}\right) = \frac{1}{2}\left(\frac{3}{2}\right)^2 = \frac{1}{2}\left(\frac{9}{4}\right) = \frac{9}{8}$

B]



$f(-2) = 4$

$f(-6) = \emptyset$ undefined

$f(0) = 0$

$f(1) = -2$

On your whiteboard...

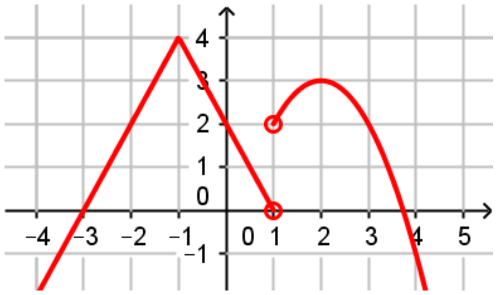
A] $f(x)=\begin{cases} -2|x+1|+4, x\le 1 \\ 3, 1<x<3 \\ -2x+8, x\ge 3 \end{cases}$

Find: $f(1)$ = 0

$f(2)$ = 3

$f(3)$ = 2

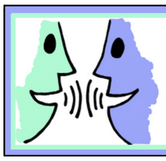
B]



Find: $f(-3)$ = 0

$f(1)$ = undefined

$f(2)$ = 3

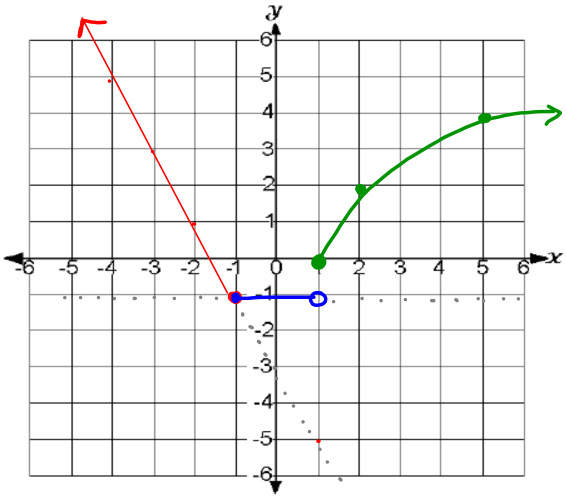


Talk & Try

In the lab, we cut and pasted pieces of functions together to create the graph.

How can you sketch the graph without scissors and glue?

$f(x)=\begin{cases} -2x-3, x<-1 \\ -1, -1\le x<1 \\ 2\sqrt{x-1}, x\ge 1 \end{cases}$

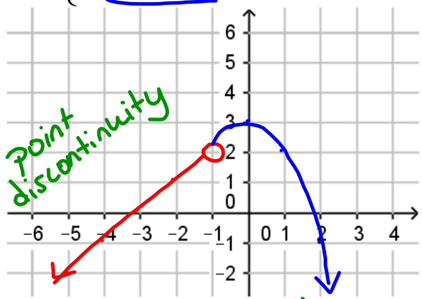


$2\sqrt{5-1}=4$

Example 2 Graphing Piecewise Functions

Graph one piece (using pencil) and erase the parts not in the domain.
Determine any open/closed circles based on the domain.
Repeat for each piece of the function.

A] $y = \begin{cases} \boxed{x+3}, x < -1 \\ \boxed{-x^2+3}, x > -1 \end{cases}$

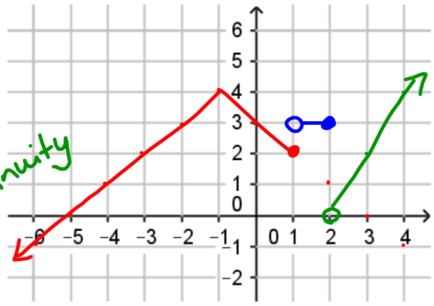


Domain: $(-\infty,-1)\cup(-1,\infty)$

Range: $(-\infty,3]$

B] $y = \begin{cases} -|x+1|+4, x \le 1 \\ 3, 1 < x \le 2 \\ 2x-4, x > 2 \end{cases}$

jump discontinuity



Domain: $(-\infty,\infty)$

Range: $(-\infty,\infty)$

Talk & Try

How can you find the <u>domain</u> and <u>range</u> WITHOUT graphing?

$f(x) = \begin{cases} 2x, x \le 0 \\ 3x-4, x < 3 \end{cases}$

Domain: Combine each subdomain

$(-\infty,0]\cup(3,\infty)$

Range: Both subfunctions are linear with positive slope, so left end points down and right end points up. Find y-values at endpoints algebraically.

$2(0)=0$
$3(3)-4=5$

$(-\infty,0]\cup(5,\infty)$

$g(x) = \begin{cases} x^2-2, x < 0 \\ 5, 0 \le x < 4 \end{cases}$

Domain: Combine each subdomain

$(-\infty,4)$

Range: The quadratic subfunction has its vertex at (0,-2) and opens up, so the range of just that piece is $(-2,\infty)$.
The constant subfunction is a horizontal line at y=5, which is already in the range of the quadratic.

$(-2,\infty)$

Example 3 Characteristics of Piecewise Functions

To find the **y-int** (0,y):

1] Find the subfunction where x=o works in the subdomain

2] Substitute 0 for x in that subfunction and evaluate

To find the **zeros** (x,0):

1] Set each subfunction equal to zero and solve for x

2] Check the zeros for each subfunction in its subdomain. If it is restricted from the domain, it is extraneous.

A] $y = \begin{cases} \boxed{x+3}, x < -1 \\ \boxed{-x^2+3}, x \geq -1 \end{cases}$

y-int:

$0 \geq -1$

$-(0)^2+3 = 3$

$(0,3)$

Zero(s):

$0 = x+3$
$-3 \quad -3$
$-3 = x$

$0 = -x^2+3$
$\sqrt{x^2} = \sqrt{3}$
$x = \sqrt{3}$
$\cancel{x = -\sqrt{3}}$ $\quad -\sqrt{3} \not\geq -1$

Example 3 Characteristics of Piecewise Functions

To find the **y-int** (0,y):

1] Find the subfunction where x=o works in the subdomain

2] Substitute 0 for x in that subfunction and evaluate

To find the **zeros** (x,0):

1] Set each subfunction equal to zero and solve for x

2] Check the zeros for each subfunction in its subdomain. If it is restricted from the domain, it is extraneous.

B] $y = \begin{cases} -|x+1|+4, x \leq 1 \\ 3, 1 < x \leq 2 \\ 2x-4, x > 2 \end{cases}$

y-int: $-|0+1|+4 = -1+4 = 3$ $(0,3)$

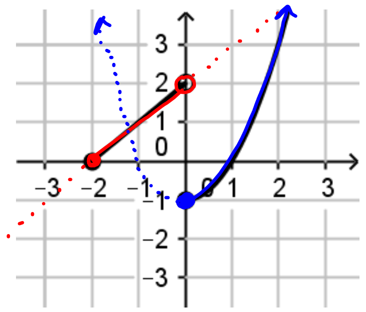
Zero(s):

Piece 1
$-|x+1|+4 = 0$
$|x+1| = 4$
$x+1 = 4 \qquad x+1 = -4$
$\cancel{x=3} \qquad x = -5$

Piece 2
None

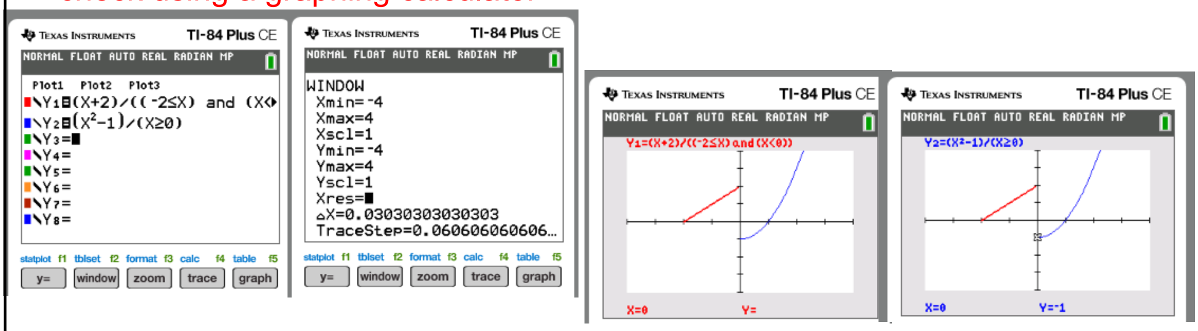
Piece 3
$2x-4=0$
$2x = 4$
$\cancel{x=2}$

Example 4 Writing Equations of Piecewise Functions Given a Graph



$$f(x) = \begin{cases} x+2, & -2 \le x < 0 \\ x^2 - 1, & x \ge 0 \end{cases}$$

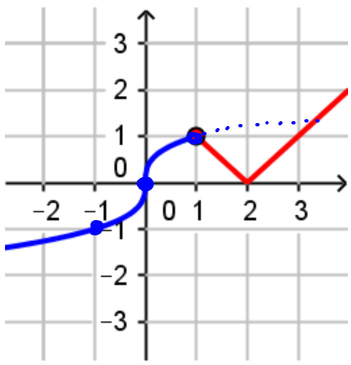
check using a graphing calculator

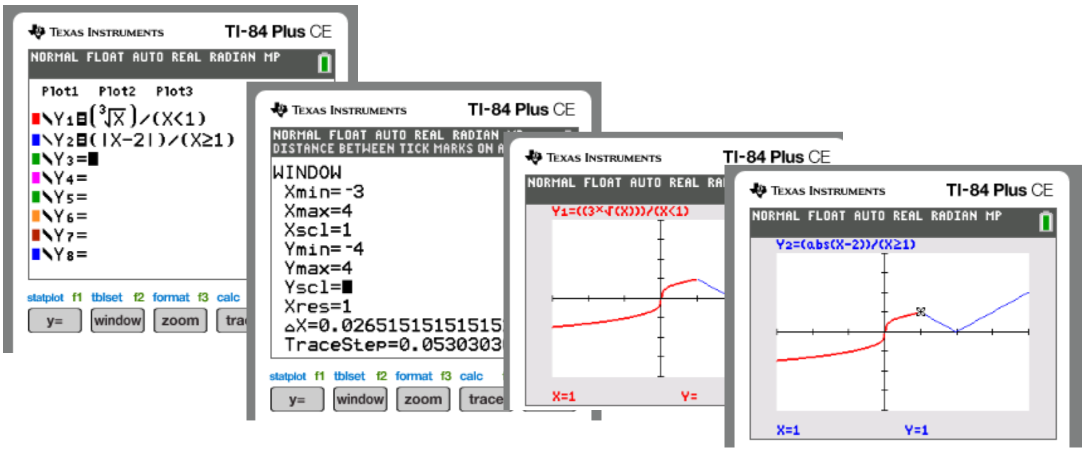


On your whiteboard...

Write the equation of the piecewise graph and check using a graphing calculator.

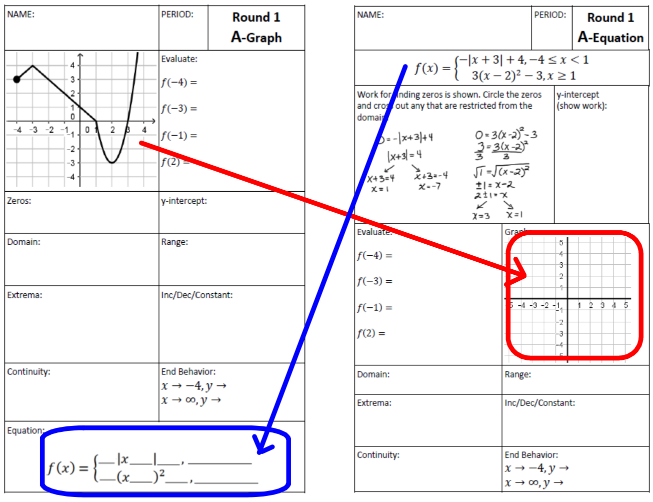
$$f(x) = \begin{cases} \sqrt[3]{x}, & x < 1 \\ |x-2|, & x \ge 1 \end{cases}$$





1] Complete your assigned problem. You are either given and graph and must write the equation for it, or you are given an equation and must graph it.

2] Find the classmate who has your corresponding problem and sit together. Compare answers and work together to identify and correct any errors as needed.



On your whiteboard...

$$f(x)=\begin{cases}-2x-3, x<-1\\ -1, -1\le x<1\\ 2\sqrt{x-1}, x\ge 1\end{cases}$$

y-int: $-1\le 0\le 1$ $(0,-1)$

Zero(s):

$-2x-3=0$
$-2x=3$
$x=-\frac{3}{2}$
$-\frac{3}{2}<-1$

$-1\neq 0$

$2\sqrt{x-1}=0$
$\sqrt{x-1}=0$
$x-1=0$
$x=1$
$1\ge 1$