Math Lab: Graphing Quadratic Equations in Standard Form

What does a tell you about the graph of $= ax^2 + bx + c$?

1] Sketch the graph of the quadratic equation given in vertex form. Label the coordinate of the vertex, y-int., and use the a-value to find the coordinates of the points one unit to the right and left of the vertex.

$$y = \frac{1}{2}(x+4)^2 - 3$$

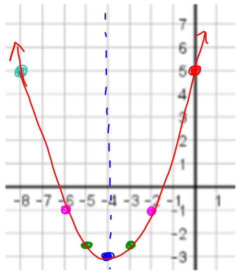
Vertex $(-4,-3)$

axis of symmetry $x=-4$

slope one unit R/L of vertex $\pm\frac{0.5}{1}$

y-int
$\frac{1}{2}(0+4)^2-3=$
$\frac{1}{2}(16)-3=$
$8-3=$
5

extra point
$\frac{1}{2}(-2+4)^2-3=$
$\frac{1}{2}(4)-3=$
$2-3=-1$



2] Rewrite the equation you just graphed in standard form. Show your work.

$$y = \frac{1}{2}(x+4)^2 - 3$$

$y=\frac{1}{2}(x+4)(x+4)-3$

$y=\frac{1}{2}(x^2+4x+4x+16)-3$

$y=\frac{1}{2}(x^2+8x+16)-3$

$y=\frac{1}{2}x^2+4x+8-3$

$y=\frac{1}{2}x^2+4x+5$

3] Sketch the graph of the quadratic equation given in vertex form. Label the coordinate of the vertex, y-int., and use the a-value to find the coordinates of the points one unit to the right and left of the vertex.

$$y = -2(x-3)^2 + 5$$

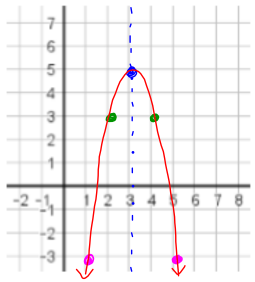
Vertex $(3,5)$

axis of symmetry $x=3$

slope one unit R/L of vertex $\pm\frac{2}{1}$

y-int
$-2(0-3)^2+5=$
$-2(9)+5=$
$-18+5=$
-13

extra point
$-2(1-3)^2+5=$
$-2(4)+5=$
$-8+5=$
-3



4] Rewrite the equation you just graphed in standard form. Show your work.

$$y = -2(x-3)^2 + 5$$

$y=-2(x-3)(x-3)+5$

$y=-2(x^2-3x-3x+9)+5$

$y=-2(x^2-6x+9)+5$

$y=-2x^2+12x-18+5$

$y=-2x^2+12x-13$

5] Does changing the form of the equation from vertex form to the standard form change the ***graph***? No

6] Does changing the form of the equation from vertex form to the standard form change the ***a*-value**? No

7] Use the graphs you sketched above as a reference to fill in the blanks below for graphing a quadratic function in standard form $y = ax^2 + bx + c$.

- When a is positive, the parabola opens up and when a is negative, the parabola opens down.
- When $|a| < 1$, the graph widens and when $|a| > 1$, the graph narrows.
- The slope from the vertex to the next guide point is $\frac{rise}{run} = \frac{\pm a}{1}$ to the right and left of the vertex.

What does c tell you about the graph of $= ax^2 + bx + c$?

Recall, that to find the y-intercept you substitute 0 for x and solve for y. Find the y-intercept of each equation below.

	What is the y-intercept?
$y = x^2 - 3$	$(0)^2 - 3 = -3$ $(0, -3)$
$y = \frac{1}{2}x^2 + 5$	$\frac{1}{2}(0)^2 + 5 = 5$ $(0, 5)$
$y = -2x^2 - 4$	$-2(0)^2 - 4 = -4$ $(0, -4)$
$y = 3x^2 + 3x + 2$	$3(0)^2 + 3(0) + 2 = 2$ $(0, 2)$
$y = -x^2 - 2x + 1$	$-(0)^2 - 2(0) + 1 = 1$ $(0, 1)$

- The y-intercept of the parabola given in the form $y = ax^2 + bx + c$ is always (0, c).

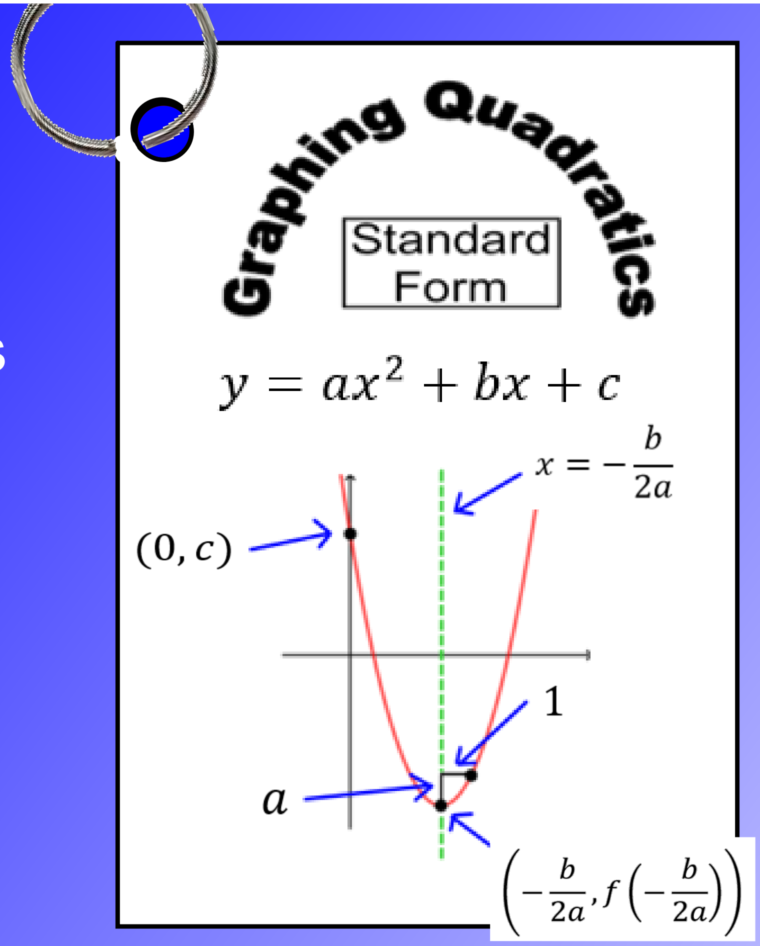
What do a and b tell you about the graph of $= ax^2 + bx + c$?

The 1st column gives you the quadratic equation in vertex form; identify the vertex in the 2nd column. The 3rd column gives you the SAME equation in standard form. Show your work for the calculations in the next two columns in the table. Use those values to answers the questions at the bottom.

Vertex Form $y = a(x - h)^2 + k$	**Vertex**	**Standard Form** $y = ax^2 + bx + c$	$-\frac{b}{2a}$ **(Show your work.)**	**Substitute** $-\frac{b}{2a}$ **in for x in the equation to find y. (Show your work.)**
$y = -(x-2)^2 + 3$	(2, 3)	$y = -x^2 + 4x - 1$ a=-1 b=4	$\frac{-4}{2(-1)} = 2$	$-(2)^2+4(2)-1 = 3$ $-4+8-1$
$y = 2(x+3)^2 - 5$	(-3, -5)	$y = 2x^2 + 12x + 13$ a=2 b=12	$\frac{-12}{2(2)} = -3$	$2(-3)^2+12(-3)+13 = -5$ $18-36+13$
$y = \frac{1}{2}(x-4)^2 + 0$	(4, 0)	$y = \frac{1}{2}x^2 - 4x + 8$ a=1/2 b=-4	$\frac{-(-4)}{2(\frac{1}{2})} = 4$	$\frac{1}{2}(4)^2-4(4)+8 = 0$ $8-16+8$
$y = -5x^2 + 10$ $-5(x+0)^2+10$	(0, 10)	$-5x^2+0x+10$ $y = -5x^2 + 10$ a=-5 b=0	$\frac{-0}{2(-5)} = 0$	$-5(0)^2+10 = 10$ $0+10$

- The equation $x = -\frac{b}{2a}$ finds the axis of Symmetry and the x-coordinate of the Vertex.
- To find the y-coordinate of the vertex, substitute x back in to $y = ax^2 + bx + c$.

Label the quadratic function with its defining characteristics based on the results of the math lab.



Fill in the blanks based on the results of the math lab.

$$y = ax^2 + bx + c$$

- The axis of symmetry is x = _-b/(2a)_.
- The vertex has x-coordinate _-b/(2a)_. Substitute x back in to find y.
- The parabola opens up when a_>_0 and opens down when a_<_0.
- The y-value of the vertex is a __minimum__ when the parabola opens up and a __maximum__ when the parabola opens down.
- The y-intercept is located at (0, _c_).
- The parabola is narrower than the parent graph $y=x^2$ if |a|_>_1 and wider if |a|_<_1.
- The slope _±a/1_ will find points on the parabola that are 1 unit to the right and left of the vertex.

Example 1 Graphing in the form $y = ax^2 + bx + c$

$$\boxed{y = 2x^2 - 8x + 6}$$

$a = 2 \quad b = -8 \quad c = 6$

Opens up or down? $a > 0$

Axis of symmetry $x = 2$

$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$

Vertex:

$y = 2(2)^2 - 8(2) + 6$

$y = 2(4) - 16 + 6$

$y = 8 - 10 = -2$

Vertex $(2, -2)$

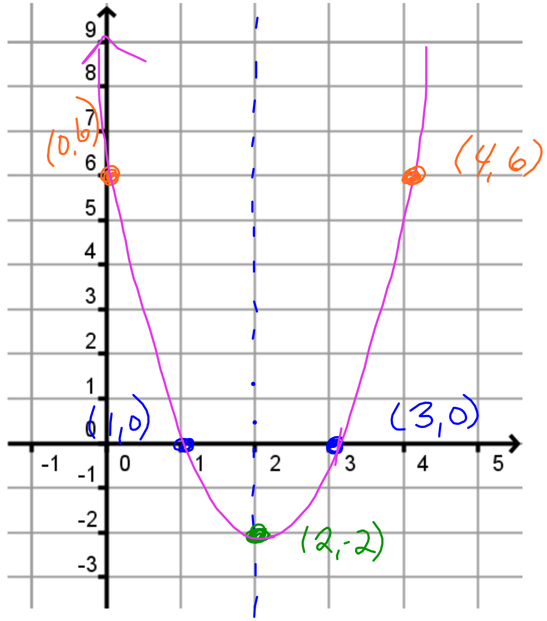
Is the vertex a max or min?

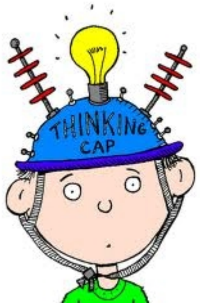
y-intercept:

$(0, c) \rightarrow (0, 6)$

Reflect the y-int over the axis of symmetry to find another point. $a = 2$

Use "a" to find points 1 unit to the right and left of the vertex.





Tell a Partner:

What information from the original equation can you use to make sure you have made a correct graph?

Does it open up or down?

Did you make a U-shape graph (parabola)?

$$y = ax^2 + bx + c$$

Did you use a as the slope to find the points ONE unit right and left of the vertex?

Did you find the axis of symmetry with -b/(2a) and substitute that back in as x to find the y of the vertex?

Did you plot (0,c) as the y-intercept?

On your whiteboard...

$$\boxed{y = x^2 - 4x + 2}$$

Axis of symmetry: x=2

$$\frac{-b}{2a} = -\frac{-4}{2(1)} = \frac{4}{2} = 2$$

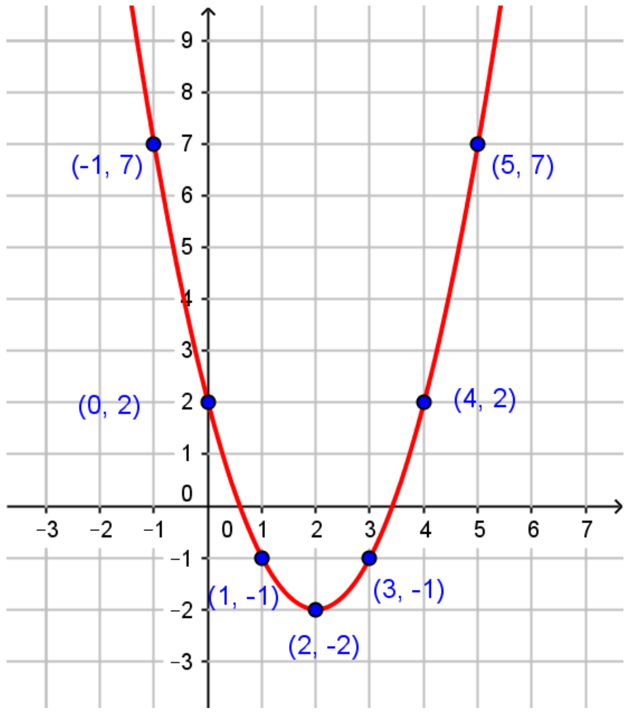
Vertex: (2,-2)

$$y = 2^2 - 4(2) + 2 = 4 - 8 + 2 = -2$$

y-intercept: (0,2)

Coordinates of four more points:

(3,-1) (1,-1) (4,2) (5,7) (-1,7)



Example 2 Graphing in the form $y = ax^2 + c$ ← no b so b=0

$$\boxed{y = \frac{-1}{2}x^2 + 3}$$

$a = -\frac{1}{2}$ $b = 0$ $c = 3$

Opens up or down? $a < 0$

Axis of symmetry:

$x = \frac{-b}{2a} = \frac{0}{2(-\frac{1}{2})} = 0$ $x = 0$

Vertex:

$y = -\frac{1}{2}(0) + 3 = 3$ → Vertex $(0,3)$

Is the vertex a max or min?

y-intercept:

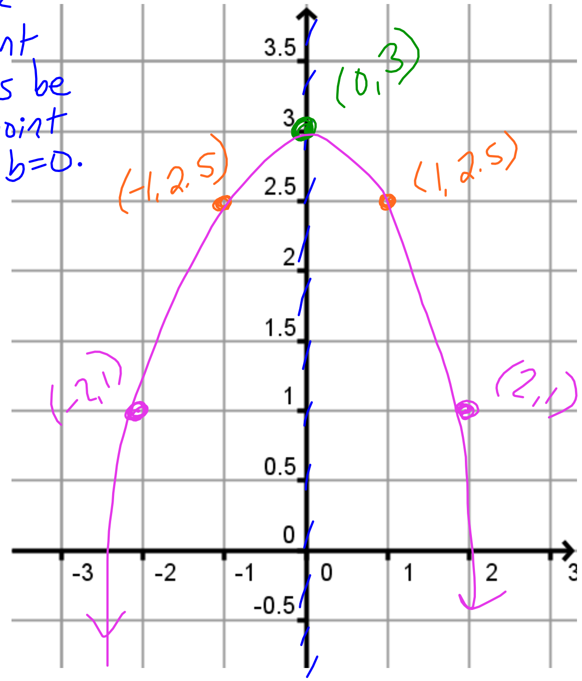
$(0,3)$

* the vertex and the y-int will always be the same point when b=0.

Use "a" to find points 1 unit to the right and left of the vertex. $a = \frac{-1}{2}$

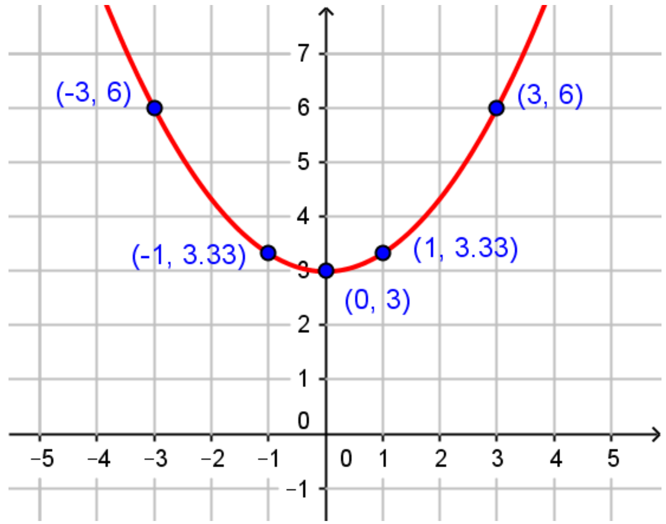
Since the vertex is also the y-intercept, you can't reflect it to find another guide point. Instead, use another value for x to find a point and then reflect it over the axis of symmetry.

$y = -\frac{1}{2}(2)^2 + 3 = 1$ $(2,1)$



On your whiteboard...

$$\boxed{y = \frac{1}{3}x^2 + 3}$$



Axis of symmetry: x=0

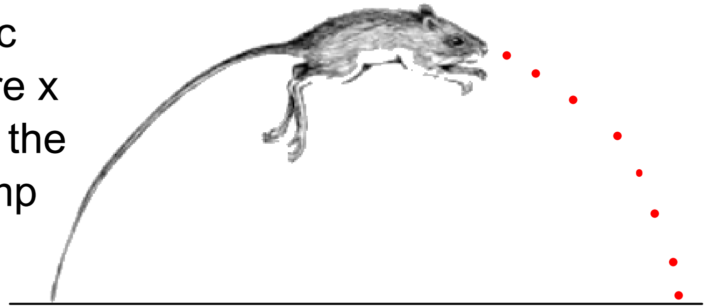
Vertex:(0,3)

y-intercept: (0,3)

Coordinates of four more points: (1,3.33) (-1,3.33) (3,6) (-3,6)

Example 3 Application problem

A woodland jumping mouse hops along a parabolic path given by the model $y = -0.2x^2 + 1.3x$, where x is the mouse's horizontal position (in feet) and y is the corresponding height (in feet). Can the mouse jump over a fence that is 3 feet high?

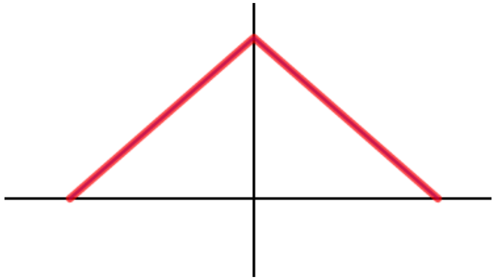


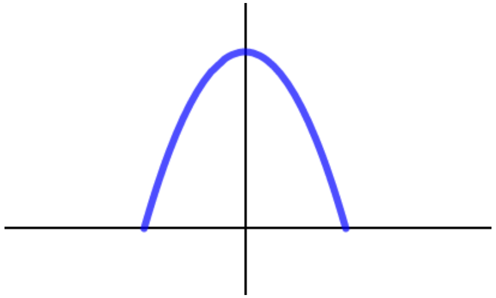
$x = \frac{-b}{2a} = \frac{-1.3}{2(-.2)} = \frac{-1.3}{-.4} = 3.25$

$y = -.2(3.25)^2 + 1.3(3.25) = 2.1125$

no, the mouse can only jump 2.1125 ft. high

When might a quadratic be a better model than an absolute value function even though both models include either a maximum or minimum value?





An absolute value model is needed when the speed is constant and is the same going up as coming down, such as a ride on an elevator that goes straight up to the top and right back down again at the same speed, or a person walking at the same speed from the door to the windows and back again.

A quadratic model is needed when the slope between points is not constant, such as kicking a ball or jumping in the air. In those examples, the acceleration due to gravity makes the object slow down as it approaches the vertex and then speed up as it falls back toward the ground.