

# Factoring

with a leading coefficient of 1

Leading coefficient:

the  $a$  in  $ax^2+bx+c$

Monomial: one term polynomial

Binomial: two term polynomial

Trinomial: three term polynomial

Factoring means to rewrite a polynomial as a product of prime factors. It is how you change from standard form...

$$ax^2+bx+c$$

into intercept form...

$$a(x-p)(x-q)$$

## Example 1 Factoring $ax^2+bx+c$ when $a=1$

$\begin{array}{c} ac \\ \times \\ b \end{array}$ 
 Step 1: If  $a \neq 1$ , then factor out the GCF if possible  
 Step 2: Find the factors of  $ac$  that add to equal  $b$   
 Step 3: Use them to rewrite in intercept form  $a(x-p)(x-q)$

A]  $x^2 + 4x - 21$

$$\begin{array}{c} -21 \\ 7 \times -3 \\ 4 \end{array}$$

$$(x+7)(x-3)$$

B]  $x^2 + 10x + 6$

$$\begin{array}{c} 6 \\ \times \\ 10 \end{array}$$

Does not factor

C]  $x^2 - 9$   $x^2 + 0x - 9$

$$\begin{array}{c} -9 \\ -3 \times 3 \\ 0 \end{array}$$

$$(x-3)(x+3)$$

D]  $2x^2 - 10x - 28$   $GCF=2$   
 $2(x^2 - 5x - 14)$

$$\begin{array}{c} -14 \\ 2 \times -7 \\ -5 \end{array}$$

$$2(x+2)(x-7)$$

E]  $2x^2 - 16x + 32$   $GCF=2$   
 $2(x^2 - 8x + 16)$

$$\begin{array}{c} 16 \\ -4 \times -4 \\ -8 \end{array}$$

$$2(x-4)(x-4)$$

$$2(x-4)^2$$

F]  $5x^2 - 25x$   $GCF=5x$   
 $5x(x-5)$   
 Already factored completely

# On your whiteboard...

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A]  $x^2 - 10x + 25$

$\begin{array}{r} 25 \\ -5 \quad -5 \\ \hline -10 \end{array}$   
 $(x-5)(x-5)$   
 $(x-5)^2$

B]  $x^2 - 25$

$x^2 + 0x - 25$   
 $\begin{array}{r} -25 \\ -5 \quad 5 \\ \hline 0 \end{array}$   
 $(x-5)(x+5)$

C]  $2x^2 + 20x - 50$  GCF=2

$2(x^2 + 10x - 25)$   
 $\begin{array}{r} -25 \\ 10 \end{array}$  Does not factor

## Example 2 Factoring $ax^2+bx+c$ when $a=1$ with big numbers

- ~~$\frac{ac}{b}$~~  Step 1: If  $a \neq 1$ , then factor out the GCF if possible  
 Step 2: Find the factors of  $ac$  that add to equal  $b$   
 Step 3: Use them to rewrite in intercept form  $a(x-p)(x-q)$

A]  $3x^2 + 60x + 153$  GCF = 3

$3(x^2 + 20x + 51)$

~~$\begin{matrix} 51 \\ 3 \times 17 \\ 20 \end{matrix}$~~

$3(x+3)(x+17)$

Plot1	Plot2	Plot3
$\sqrt{Y_1} = 51/X$		
$\sqrt{Y_2} =$	X	Y1
$\sqrt{Y_3} =$	0	ERROR
$\sqrt{Y_4} =$	1	51
$\sqrt{Y_5} =$	2	25.5
$\sqrt{Y_6} =$	3	17
$\sqrt{Y_7} =$	4	12.75
	5	10.2
	6	8.5

Press + for  $\Delta Tbl$

3+17=20

B]  $x^2 - 21x - 72$

~~$\begin{matrix} -72 \\ 3 \times -24 \\ -21 \end{matrix}$~~

$(x+3)(x-24)$

Plot1	Plot2	Plot3
$\sqrt{Y_1} = -72/X$		
$\sqrt{Y_2} =$	X	Y1
$\sqrt{Y_3} =$	0	ERROR
$\sqrt{Y_4} =$	1	-72
$\sqrt{Y_5} =$	2	-36
$\sqrt{Y_6} =$	3	-24
$\sqrt{Y_7} =$	4	-18
	5	-14.4
	6	-12

Press + for  $\Delta Tbl$

3 + (-24) = -21

On your whiteboard...

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Factor  $x^2 - 19x + 78$

$$\begin{array}{ccc} & 78 & \\ -6 & \times & -13 \\ & -19 & \end{array}$$
$$(x-6)(x-13)$$

Plot1	Plot2	Plot3
$Y_1 = 78/X$		
$Y_2 =$		
$Y_3 =$		
$Y_4 =$		
$Y_5 =$		
$Y_6 =$		
$Y_7 =$		

X	Y <sub>1</sub>
-8	-9.75
-7	-11.14
-6	-13
-5	-15.6
-4	-19.5
-3	-26
-2	-39

X = -8

$$-6 + (-13) = -19$$

### Example 3 Using factoring to solve equations

1. Rewrite in standard form so the quadratic equation is equal to zero.
2. Factor completely.
3. Set each factor equal to zero and solve for x.

\*Repeat the zero product property says anything times 0 equals 0.

A)  $x(x+2) = 6+x$

$$\begin{array}{r} x^2 + 2x = 6 + x \\ -x \quad -x \\ \hline \end{array}$$

$$\begin{array}{r} x^2 + x = 6 \\ -6 \quad -6 \\ \hline \end{array}$$

$$x^2 + x - 6 = 0$$

standard form = to 0 so now you can factor

$$\begin{array}{r} -6 \\ 3 \times -2 \\ 1 \end{array} \quad (x+3)(x-2) = 0$$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline x=-3 \end{array}$$

$$\begin{array}{r} x-2=0 \\ +2 \quad +2 \\ \hline x=2 \end{array}$$

check!

$$x(x+2) = 6+x$$

$$-3(-3+2) = 6+(-3)$$

$$-3(-1) = 6-3$$

$$3=3 \checkmark$$

$$2(2+2) = 6+2$$

$$2(4) = 8$$

$$8=8 \checkmark$$

B)  $(x+3)^2 = -2(x+3)$

$$(x+3)(x+3) = -2(x+3)$$

$$\begin{array}{r} x^2 + 6x + 9 = -2x - 6 \\ +6 \quad +6 \\ \hline \end{array}$$

$$\begin{array}{r} x^2 + 6x + 15 = -2x \\ +2x \quad +2x \\ \hline \end{array}$$

$$x^2 + 8x + 15 = 0$$

standard form = to 0 so now you can factor

$$\begin{array}{r} 15 \\ 5 \times 3 \\ 8 \end{array}$$

$$(x+5)(x+3) = 0$$

$$\begin{array}{r} x+5=0 \\ -5 \quad -5 \\ \hline x=-5 \end{array}$$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline x=-3 \end{array}$$

check!

$$(x+3)^2 = -2(x+3)$$

$$\begin{array}{l} (-5+3)^2 = -2(-5+3) \\ (-2)^2 = -2(-2) \\ 4=4 \checkmark \end{array}$$

$$\begin{array}{l} (-3+3)^2 = -2(-3+3) \\ (0)^2 = -2(0) \\ 0=0 \checkmark \end{array}$$

On your whiteboard...

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Solve the quadratic equation by factoring.

$$x^2 - 16 = -2(x - 16)$$

$$\begin{array}{r} x^2 - 16 = -2x + 32 \\ -32 \quad -32 \\ \hline \end{array}$$

$$\begin{array}{r} x^2 - 48 = -2x \\ +2x \quad +2x \\ \hline \end{array}$$

$$x^2 + 2x - 48 = 0$$

$$\begin{array}{r} -48 \\ 8 \times -6 \\ \hline 2 \end{array}$$

$$(x+8)(x-6) = 0$$

$$x+8=0$$

$$x = -8$$

$$x-6=0$$

$$x = 6$$

Check!

$$(-8)^2 - 16 = -2(-8 - 16)$$

$$64 - 16 = -2(-24)$$

$$48 = 48$$

✓

$$6^2 - 16 = -2(6 - 16)$$

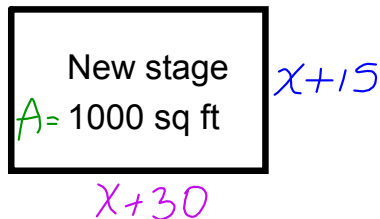
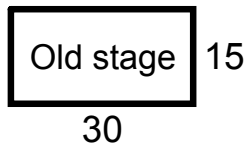
$$36 - 16 = -2(-10)$$

$$20 = 20$$

✓

## Example 4 Application problem

The old stage at the high school was 15x30 feet. It was expanded by adding the same distance,  $x$ , to the width and length of the old stage dimensions. The new stage has a total area of 1000 sq feet. What are the dimensions of the new stage?



$$\text{Area} = \text{Length}(\text{width})$$

$$1000 = (x+30)(x+15)$$

$$\begin{array}{r} 1000 = x^2 + 45x + 450 \\ -1000 \qquad \qquad -1000 \\ \hline 0 = x^2 + 45x - 550 \end{array}$$

$$\begin{array}{r} -550 \\ 55 \times -10 \\ 45 \end{array}$$

$$0 = (x+55)(x-10)$$

$x = -55$   
Not useful  
since length  
can't be negative.

$x = 10$   
Substitute back  
in for  $x$  to  
find dimensions

$$\begin{array}{l} \text{Length: } 10+30 = 40 \\ \text{Width: } 10+15 = 25 \end{array}$$

New stage is  
25 x 40 feet



# Basic Factoring Connect-4

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## Rules:

- One person gets a blue pencil and the other gets a red pencil.
- The person with the red pencil goes first. Pick a square and solve the quadratic equation by factoring. Write the factored form and the solutions in the box using the red pencil. Have your partner check your work. If it is correct, you win that square. If it is wrong, your partner wins that square and circles it in his/her color.
- The person with the blue pencil goes next. Then keep taking turns.
- The game is over when someone gets four in row (across, up/down, or diagonal).

